Kenmore-Town of Tonawanda UFSD

We educate, prepare, and inspire all students to achieve their highest potential

Grade 2 Module 2
Parent Handbook

Addition and Subtraction of Length Units

In this module, we will be exploring the ruler, estimating and measuring lengths using various tools and units, and finally, relating addition and subtraction to length.

What Came Before this Module:
We practiced making sums and differences to the number 20

What Comes After this Module:
We will begin work with the base-10 place value system

Key Words to Know

Endpoint: Where something ends, where measurement begins

Hash mark: The marks on a ruler or other measurement tool

Number Line: A line marked at evenly spaced intervals

Estimate: An approximation of the value of a quantity or number

Tape Diagram: See back of this sheet!

Common Words:

Length
Combine
Difference
Meter
Height
Compare
Centimeter

How you can help at home:

- Ask questions that encourage your student to estimate lengths of household items
- Continue to review adding and subtracting up to 20
- Practice measuring lengths longer than a ruler by marking and measuring from a mark

Key Common Core Standards:
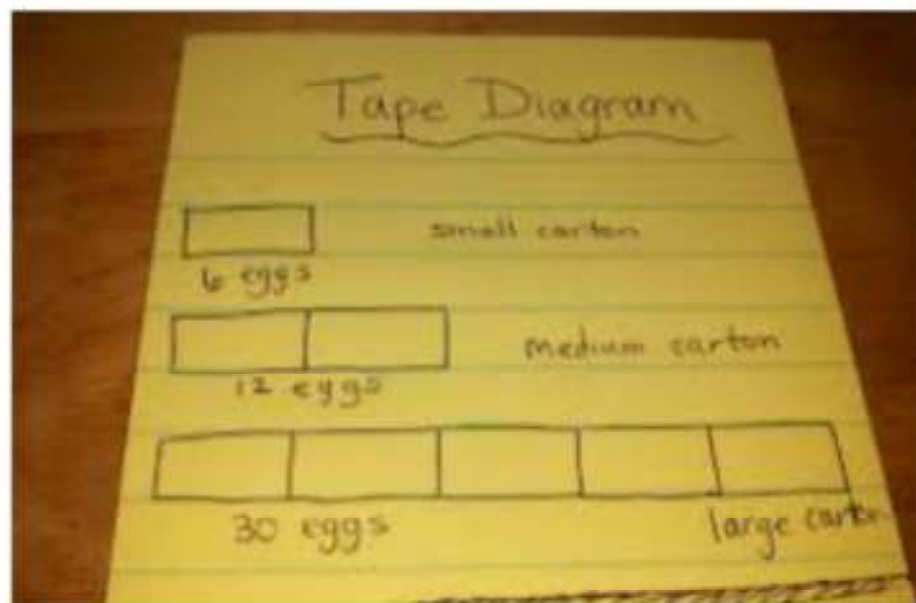
- *Relate addition and subtraction to length*

Examples:

- Line A is 4 cm long, and Line B is 7 cm long. Together, Lines A and B measure _____ cm.

- In the example above, how much shorter is Line A than Line B?

- *Measure and estimate lengths in standard and non-standard units*

Examples:

- How many centimeter cubes long is my pencil?

- How many Lego-pieces long is this bracelet?

Spotlight on Math Models:

Tape Diagram

You will often see this mathematical representation in *A Story of Units.*

A Story of Units has several key mathematical "models" that will be used throughout a student's elementary years.

The tape diagram is a powerful model that students can use to solve various kinds of problems. In second grade, you will often see this model as an aid to addition and subtraction problems. Tape diagrams are also called "bar models" and consist of a simple bar drawing that students make and adjust to fit a word problem. They then use the drawing to discuss and solve the problem.

As students move through the grades, tape diagrams provide an essential bridge to algebra. Below is a sample word problem from Module 2 solved using a tape diagram to show the parts of the problem.

Sample Problem from Module 2:
(Example taken from Module 2, Lesson 7)

Natalia, Chloe, and Lucas are making clay snakes. Natalia's snake is 16 centimeters. Chloe's snake is 5 centimeters shorter than Natalia's. How long is Chloe's snake?

Lucas's snake is 3 centimeters longer than Chloe's snake. Who has the longest snake: Natalia, Lucas, or Chloe?
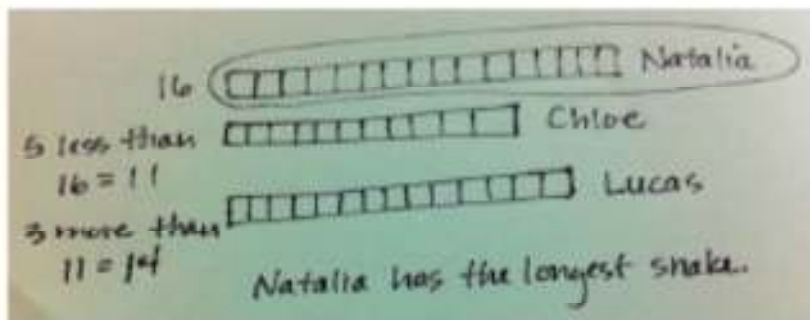
Addition and Subtraction of Length Units

OVERVIEW

In this 12-day Grade 2 module, students engage in activities designed to deepen their conceptual understanding of measurement and to relate addition and subtraction to length. Their work in Module 2 is exclusively with metric units in order to support place value concepts. Customary units are introduced in Module 7.

Topic A opens with students exploring concepts related to the centimeter ruler. In the first lesson, they are guided to connect measurement with physical units as they find the total number of unit lengths by laying multiple copies of centimeter cubes (physical units) end-to-end along various objects. Through this, students discover that to get an accurate measurement, there must be no gaps or overlaps between consecutive length units.

Next, students measure by iterating with one physical unit, using the *mark and advance* technique, also known as *mark and move forward*. Students then repeat the process by laying both multiple copies and a single cube along a centimeter ruler. This helps students create a mental benchmark for the centimeter. It also helps them realize that the distance between 0 and 1 on the ruler indicates the amount of space already covered. Hence 0, not 1, marks the beginning of the total length. Students use this understanding to create their own centimeter rulers using a centimeter cube and the mark and advance technique. Topic A ends with students using their unit rulers to measure lengths (**2.MD.1**), thereby connecting measurement with a ruler.

Students build skill in measuring using centimeter rulers and meter sticks in Topic B. They learn to see that a length unit is not a cube, or a portion of a ruler (which has width), but is a segment of a line. By measuring a variety of objects, students build a bank of known measurements or benchmark lengths, such as a doorknob being a meter from the floor, or the width of a finger being a centimeter. Then, students learn to estimate length using knowledge of previously measured objects and benchmarks. This enables students to internalize the mental rulers1 of a centimeter or meter, empowering them to mentally iterate units relevant to measuring a given length (**2.MD.3**). The knowledge and experience signal that students are determining which tool is appropriate to make certain measurements (**2.MD.1**).

In Topic C, students measure and compare to determine how much longer one object is than another **(2.MD.4)**. They also measure objects twice using different length units, both standard and non-standard, thereby developing their understanding of how the total measurement relates to the size of the length unit **(2.MD.2).**   Repeated experience and explicit comparisons help students recognize that the smaller the length unit, the larger the number of units, and the larger the length unit, the smaller the number of units.

The module culminates as students relate addition and subtraction to length. They apply their conceptual understanding to choose appropriate tools and strategies, such as the ruler as a number line, benchmarks for estimation, and tape diagrams for comparison, to solve word problems (**2.MD.5**, **2.MD.6**). The problems progress from concrete (i.e., measuring objects and using the ruler as a number line to add and subtract) to abstract (i.e., representing lengths with tape diagrams to solve *start unknown* and two-step problems).

Terminology

New or Recently Introduced Terms

- Benchmark (e.g., "round" numbers like multiples of 10)
- Endpoint (point where something begins or ends)
- Estimate (an approximation of a quantity or number)
- Hash mark (marks on a ruler or other measurement tool)
- Meter (standard unit of length in the metric system)
- Meter stick or strip (tool used to measure length)
- Number line
- Overlap (extend over, or cover partly)
- Ruler (tool used to measure length)
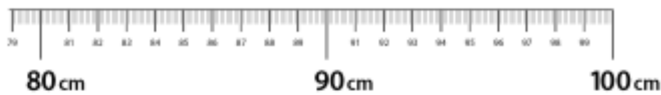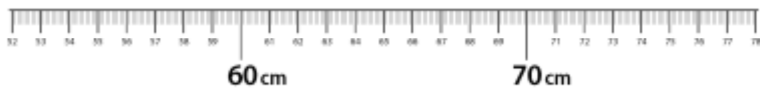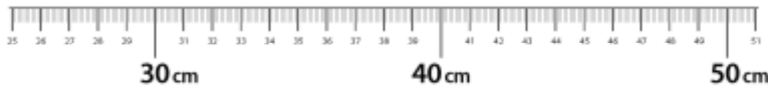
Number line

Familiar Terms and Symbols

- Centimeter (standard length unit within the metric system)
- Combine (join or put together)
- Compare (specifically using direct comparison)
- Difference (to find the difference between two numbers, subtract the smaller number from the greater number)
- Height (vertical distance measurement from bottom to top)
- Length (distance measurement from end to end; in a rectangular shape, length can be used to describe any of the four sides) Length unit (e.g., centimeters, inches)

Suggested Tools and Representations

- Centimeter cubes
- Centimeter rulers
- Large and small paper clips
- Meter stick
- Paper meter strips (Lesson 6 Template)
- Personal white boards
- Tape diagram

Tape Diagrams

A tape diagram is a way for students to visually represent a mathematical problem. It helps students to break down and make sense of a word problem. It provides students access to selecting the appropriate operation as they visualize the relationships between the quantities. Tape diagrams enable students to solve problems efficiently. In Grade 2 Module 2 students will utilize tape diagrams to solve addition and subtraction problems. Several examples follow:

Carol's ribbon is 76 centimeters long. Alice's ribbon is 1 meter long. How much longer is Alice's ribbon than Carol's?
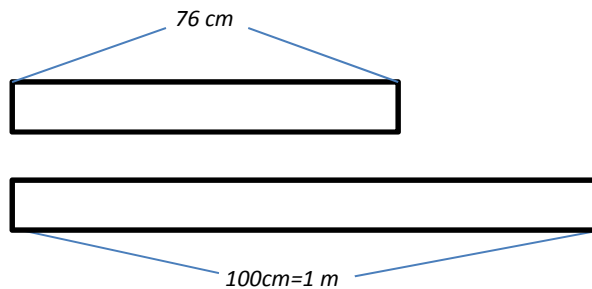
76 cm

76 + ___ = 100

76 + 4 = 80   80 + 20 = 100

4 + 20 = 24

100cm=1 m

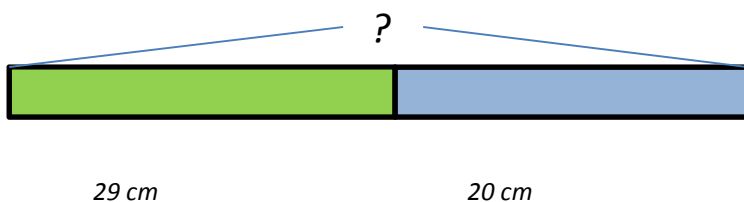*76 + 24 = 100*

Alice's ribbon is 24 cm longer than Carol's.

There is 29 cm of green ribbon and 20 cm of blue ribbon. Find the total length of the blue and green ribbons.

?

29 cm   *20 cm*

29 cm + 20 cm = ?

20 + 9   + 20 = ?

20 + 20 = 40   40 + 9 = 49

The total length of the ribbons is 49 cm.

Watch a short video example of a student using a tape diagram at:

https://www.youtube.com/watch?v=GT4fEhfE_8E

Additional Resources for Parents can be found at:

http://greatminds.net/parents

Grade 2 Module 2 Topic A

Understand Concepts About the Ruler

Focus Standard:

2.MD.1   Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.

Instructional Days Recommended: 3

Topic A begins with an exploration of concepts about the ruler. In Lesson 1, students relate length to physical units by measuring various objects with multiple centimeter cubes, creating a mental benchmark for the centimeter. In Lesson 2, they apply their knowledge of using centimeter cubes to measure by moving from repeated physical units to the iteration of one physical unit. This enables them to internalize their understanding of a length unit as the amount of space between one end of the cube to the other (or space between hash marks). Thus, they begin moving from the concrete to the conceptual. Finally, in Lesson 3, students apply knowledge of known measurements to create unit rulers using one centimeter cube. This deepens the understanding of distance on a ruler and the ruler as a number line.

Lesson 1

Objective: Connect measurement with physical units by using multiple copies of the same physical unit to measure.

Homework Key

1. 4

2. 5

3. 4

4. 10

5. 79 cm

6. 24 cm
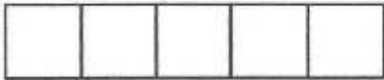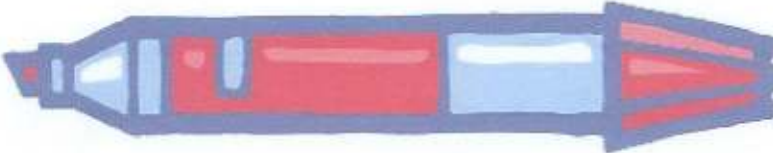
7. 29 cm

Lesson 1 (continued)

Work Samples

Count each centimeter cube to find the length of each object.

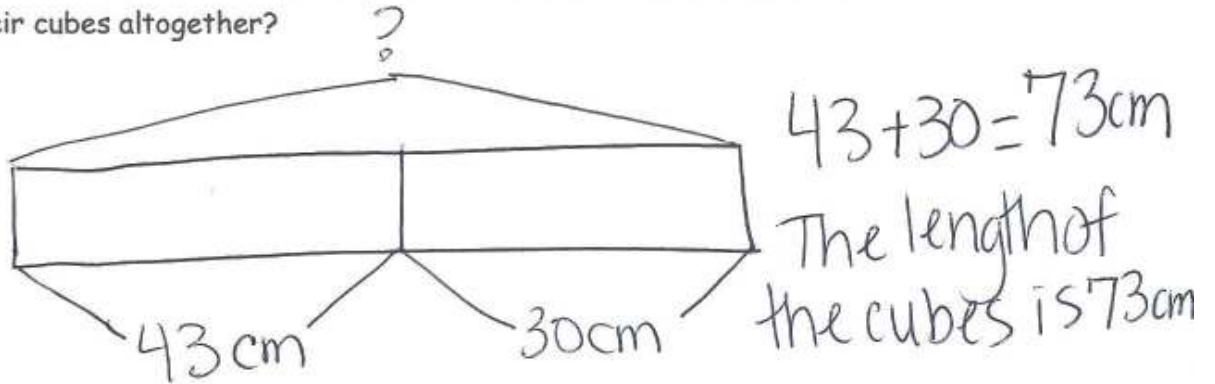1. The crayon is ___4___ centimeter cubes long.

2. The pencil is ___5___ centimeter cubes long.

3. The clothespin is ___4___ centimeter cubes long.

4. The length of the marker is ___10___ centimeter cubes.

5. Richard has 43 centimeter cubes. Henry has 30 centimeter cubes. What is the length of their cubes altogether?

?

43 cm   30cm

$43 + 30 = 73$cm

The length of the cubes is 73cm

Lesson 2

Objective: Use iteration with one physical unit to measure.

Homework Key

1. 5
2. 8
3. 7

4. a. 9
    b. 6
    c. 4
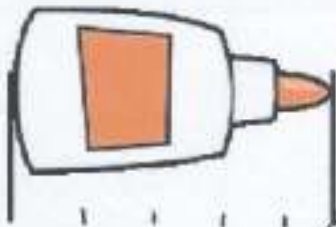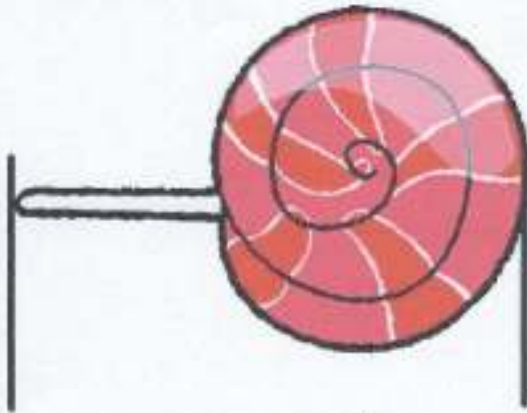   d. Red
    e. Yellow
    f. 19

Work Samples

Use the centimeter square at the bottom of the next page to measure the length of each object. Mark the endpoint of the square as you measure.

1. The picture of the glue is about __5__ centimeters long.

2. The picture of the lollipop is about __8__ centimeters long.

3. The picture of the scissors is about __7__ centimeters long.

4. Samantha used a centimeter cube and the mark and move forward strategy to measure these ribbons. Use her work to answer the following questions.

Red Ribbon

9cm

Blue Ribbon

6cm

Yellow Ribbon

4cm

?
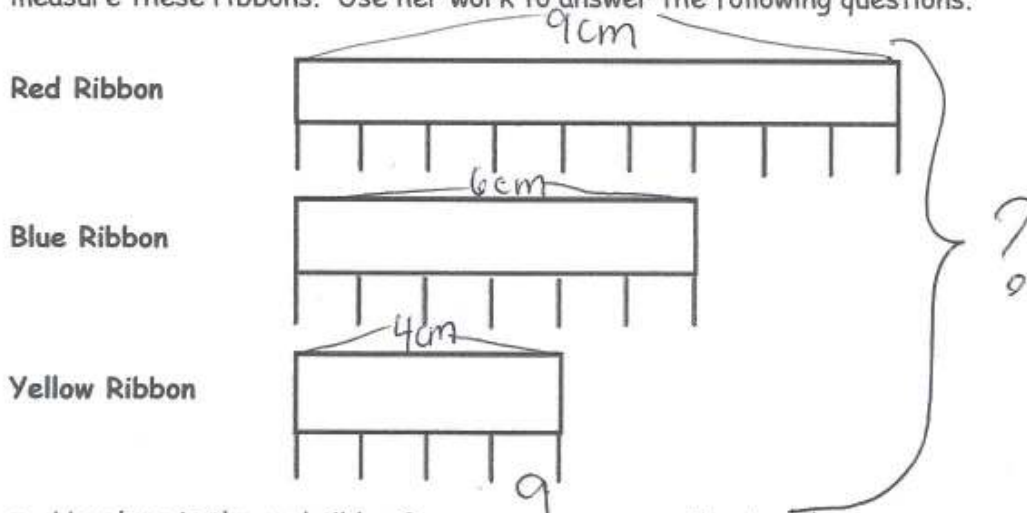
a. How long is the red ribbon? _____9_____ centimeters long.

b. How long is the blue ribbon? _____6_____ centimeters long.

c. How long is the yellow ribbon? _____4_____ centimeters long.

d. Which ribbon is the longest? (Red)   Blue   Yellow

e. Which ribbon is the shortest? Red   Blue   (Yellow)

$$9+1=10$$
$$10+5+4=$$
$$19cm$$

f. The total length of the ribbons is __19__ centimeters.

$$9+6+4$$

Lesson 3

Objective: Apply concepts to create unit rulers and measure lengths using unit rulers.

Homework Key

1. 3

2. 11

3. 8

4. Side A = 3 cm; Side B = 6 cm; Side C = 5 cm; Side D = 6 cm

   a. Side B

   b. 3

   c. 2

   d. 12

   e. 20

Work Samples

Measure the lengths of the objects with the centimeter ruler you made in class.

1. The picture of the fish is __3__ cm long.

2. The picture of the fish tank is __13__ cm long.

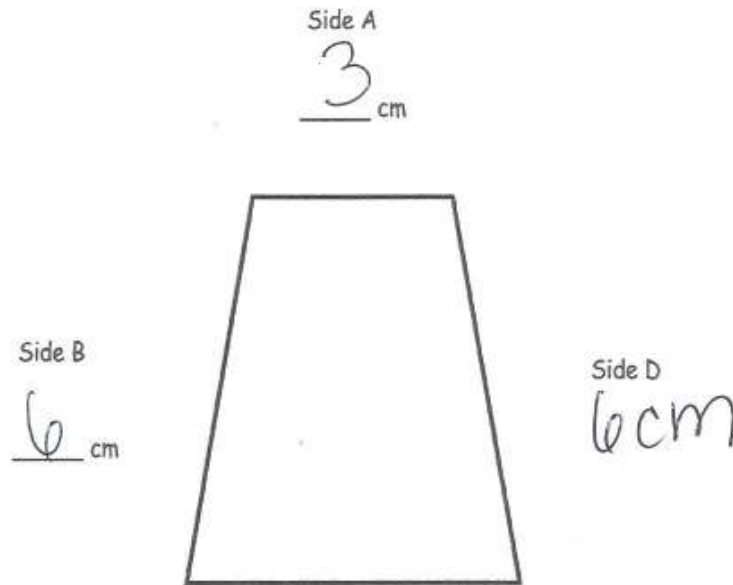3. The picture of the fish tank is __10__ cm longer than the picture of the fish.

   13 - 3 = 10

Lesson 3 (continued)

4. Measure the lengths of Sides A, B, and C. Write each length on the line.

Side A

$\underline{3}$ cm

Side B

$\underline{6}$ cm

Side D

6 cm

Side C

$\underline{5}$ cm

a. Which side is the longest?   Side A   (Side B)   Side C

b. How much longer is Side B than Side A? __3__ cm longer   6 - 3 = 3

c. How much shorter is Side A than Side C? __2__ cm shorter   5 - 3 = 2

d. Sides B and D are the same length.
   What is the length of Sides B and D together? __12__ cm   6 + 6 = 12

e. What is the total length of all four sides of this figure? __20__ cm   6 + 3 + 6 + 5 =

Grade 2 Module 2 Topic B

Measure and Estimate Length Using Different Measurement Tools

Focus Standards:

2.MD.1   Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.

2.MD.3   Estimate lengths using units of inches, feet, centimeters, and meters.

Instructional Days Recommended: 2

In Lesson 4, students begin to use centimeter rulers, meter sticks, and meter tapes to measure various objects. Through the practice of measuring various items and learning mental benchmarks for measurement, students organically develop estimation skills in Lesson 5. They also develop their skills for selecting an appropriate measuring tool by referencing prior knowledge of objects they have already measured, as well as by using mental benchmarks.

Lesson 4

Objective: Measure various objects using centimeter rulers and meter sticks.

Homework Key

1. a. cm
   b. m
   c. cm
   d. cm
   e. m
   f. m
   g. cm
   h. cm
   i. m
   j. m

2. a. m
   b. m
   c. cm
   d. m
   e. cm

3. a. Triangle A: 3
      Semi-circle C: 5
      Rectangle E: 4
      Square B: 3
      Hexagon D: 4
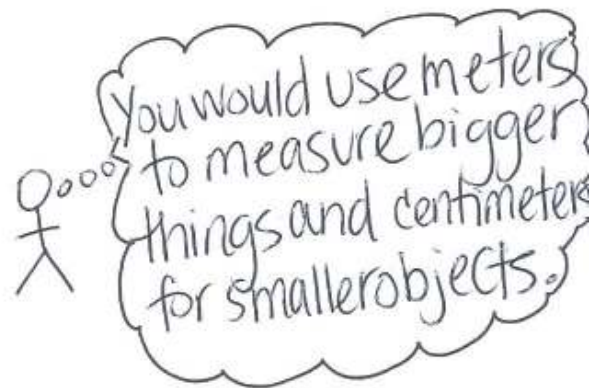   b. Answers will vary.

Lesson 4 (continued)

Work Samples

1. Circle cm (centimeter) or m (meter) to show which unit you would use to measure the length of each object.

   a. Length of a marker   (cm) or m

   b. Length of a school bus   cm or (m)

   c. Length of a laptop computer   (cm) or m

   d. Length of a highlighter marker   (cm) or m

   e. Length of a football field   cm or (m)

   f. Length of a parking lot   cm or (m)

   g. Length of a cell phone   (cm) or m

   h. Length of a lamp   (cm) or m

   i. Length of a supermarket   cm or (m)

   j. Length of a playground   cm or (m)

2. Fill in the blanks with **cm** or **m**.

   a. The length of a swimming pool is 25 __m__.

   b. The height of a house is 8 __m__.

   c. Karen is 6 __cm__ shorter than her sister.

   d. Eric ran 65 __m__ down the street.

   e. The length of a pencil box is 3 __cm__ longer than a pencil.

You would use meters to measure bigger things and centimeters for smaller objects.

Lesson 5

Objective: Develop estimation strategies by applying prior knowledge of length and using mental benchmarks.

Homework Key

1. Answers will vary.

2. a. 3 m
   b. 14 cm
   c. 17 cm
   d. 16 cm
   e. 50 m

3. a. 6
   b. 1
   c. 15
   d. 9
   e. 4

Work Samples

1. Name five things in your home that you would measure in meters.
   Estimate their length.
   *Remember, the length from a doorknob to the floor is about 1 meter.

Item	Estimated Length
a. carpet	4 meters
b. pool	25 meters
c. bathtub	2 meters
d. bed	3 meters
e. couch	3 meters

2. Choose the best length estimate for each object.

   a. Whiteboard   (3 m)   or   45 cm

   b. Banana   (14 cm)   or   30 cm

   c. DVD   25 cm   or   (17 cm)

   d. Pen   (16 cm)   or   1 m

   e. Swimming pool   (50 m)   or   150 cm

Grade 2 Module 2 Topic C

Measure and Compare Lengths Using Different Length Units

Focus Standards:

2.MD.1   Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.

2.MD.2   Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.

2.MD.4   Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.

Instructional Days Recommended: 2

In Topic C, students use different length units to measure and compare lengths. They practice applying their knowledge of centimeters and meters to choose an appropriate measurement tool in Lesson 6. Students discover that there is a relationship between unit size and measurement when they measure one object twice using different length units. They learn that the larger the unit, the fewer number of units in a given measurement. In Lesson 7, students continue to measure and compare lengths using standard and non-standard length units. At this point, students are prepared to explicitly compare different non-standard length units and can make inferences about the relative size of objects.

Lesson 6

Objective: Measure and compare lengths using centimeters and meters.

Homework Key

1.	a.	4	3.	a.	25	
	b.	26		b.	15	
2.	a.	8		c.	20	
	b.	9		d.	245	
	c.	5	4.	a.	10	
	d.	22		b.	46	
	e.	3		c.	46	
	f.	1		d.	35	
	g.	4		e.	49	
	h.	7	5.	a.	33 cm	
				b.	63 cm	
			6.		56 cm	

Work Samples

Measure each set of lines in centimeters, and write the length on the line. Complete the comparison sentences.

1. Line A _____ 14cm _____

    Line B _____ 10cm _____ ?

    a. Line A is about __4__ cm longer than line B.   14−10=4

    b. Line A and B are about __24__ cm combined.   14+10=24

2. Line X _____ 8cm _____

    Line Y _____ 9 cm _____

    Line Z _____ 5cm _____

    a.   Line X   Line Y   Line Z

    __8__ cm   __9__ cm   __5__ cm

    b. Lines X, Y, and Z are about __22__ cm combined.

    c. Line Z is about __3__ cm shorter than Line X.

    d. Line X is about __1__ cm shorter than Line Y.

    e. Line Y is about __4__ cm longer than Line Z.

    f. Line X doubled is about __7__ cm longer than line Y.

Lesson 7

Objective: Measure and compare lengths using standard metric length units and non-standard length units; relate measurement to unit size.

Homework Key

1. a. 3
   b. 9
   c. 6; 18

2. a. 4
   b. 16
   c. 2
   d. 7
   e. 2
   f. 2

3. Lines measuring 9 cm and 12 cm drawn
   a. 3
   b. 4
   c. 3
   d. 1
   e. 7
   f. 21

4. a. 100
   b. Line X
   c. 11

5. a. 200
   b. 3
   c. Line Q
   d. 100 cm or 1 m

6. a. 4
   b. 6
   c. Answers will vary.

Work Samples

Use a centimeter ruler and paper clips to measure and compare lengths.

1. |—+—+—+—+—+—+—+—| Line Z

   a. Line Z

   __3__ paper clips   __9__ cm

   b. Line Z doubled would measure about __6__ paper clips or about __18__ cm long.

   3+3   9+9

Grade 2 Module 2 Topic D

Relate Addition and Subtraction to Length

Focus Standards:

2.MD.5   Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.

2.MD.6   Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, …, and represent whole-number sums and differences within 100 on a number line diagram.

Instructional Days Recommended: 3

In Topic D, students relate addition and subtraction to length. They apply their conceptual understanding to choose appropriate tools and strategies (e.g., the ruler as a number line, benchmarks for estimation, tape diagrams for comparison) to solve word problems (**2.MD.5**, **2.MD.6**).

Students had their first experience creating and using a ruler as a number line in Topic A. Now, in Lesson 8, students solve addition and subtraction word problems using the ruler as a number line. This concept is reinforced and practiced throughout the module in fluency activities that involve using the meter strip for counting on and counting back and is incorporated into the accompanying Problem Sets. In Lesson 9, students progress from concrete to abstract by creating tape diagrams to represent and compare lengths. Lesson 10 culminates with students solving two-step word problems involving measurement using like units.

The sample homework responses contained in this manual are intended to provide insight into the skills expected of students and instructional strategies used in Eureka Math.
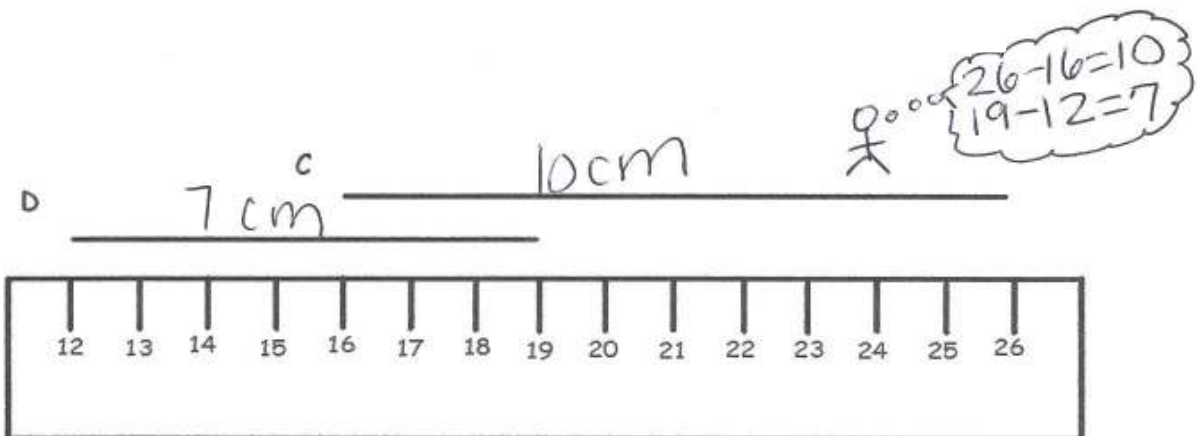
Lesson 8

Objective: Solve addition and subtraction word problems using the ruler as a number line.

Homework Key

1. a. 10
   b. 7
   c. 17
   d. 3; longer
2. 18 m

3. a. 3 written in each box
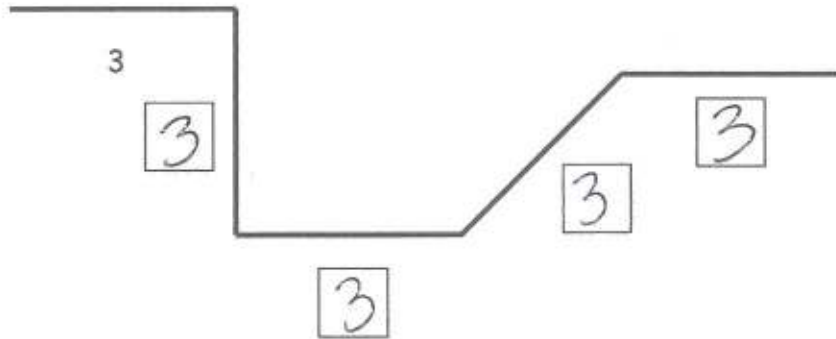   b. 15
   c. 2
4. 27 cm

Work Samples

1.

$26-16=10$
$19-12=7$

C
10cm

D
7 cm

| 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 |

a. Line C is __10__ cm.

b. Line D is __7__ cm.

c. Lines C and D are __17__ cm.

d. Line C is __3__ cm (longer/shorter) than Line D.

3. All of the parts of the path below are equal length units.
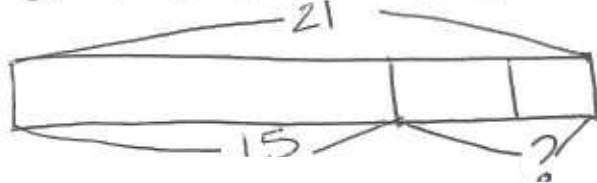
3

3

3

3

3

3

a. Fill in the empty boxes with the lengths of each side.

b. The path is __15__ length units long. $3+3+3+3+3$

c. How many more parts would you need to add for the path to be 21 length units long?

__2__ parts $15+3+3=21$

-21

-15 $-?$

Lesson 9

Objective: Measure lengths of string using measurement tools, and use tape diagrams to represent and compare lengths.

Homework Key

1. Answers will vary.
   a. Answers will vary.
   b. Answers will vary.
   c. Answers will vary.

2. a. 9
   b. 11
   c. 20
   d. 2; shorter

3. a. 40
   b. 60

4. a. 98
   b. Steven
   c. Tape diagram showing that 98 cm is 23 cm longer than 75 cm

Work Samples

1. Mia completed the chart by first estimating the measurement around three objects in her house and then finding the actual measurement with her meter strip.

Object Name	Estimated Measurement in Centimeters	Actual Measurement in Centimeters
Orange	40 cm	36 cm
Mini Basketball	30 cm	41 cm
Bottom of a glue bottle	10 cm	8 cm

a. What is the difference between the longest and shortest measurements?

___33___ cm

$$41-8=33$$
$$31 \; 10$$
$$10-8=2$$
$$31+2=33$$

b. Draw a tape diagram comparing the measurements of the orange and the bottom of the glue bottle.

Orange | 36cm |
Glue Bottle | 8cm | ___?___

$$36-8=28cm$$
$$10 \; 26$$
$$10-8=2$$
$$26+2=28$$
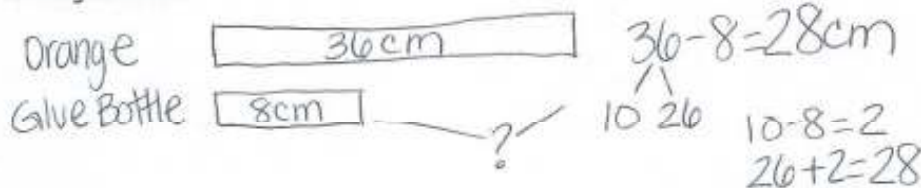
c. Draw a tape diagram comparing the measurements of the basketball and the bottom of the glue bottle.

Basketball | 41 cm |
Glue Bottle | 8cm | ___?___

$$41-8=33cm$$
$$10 \; 31$$
$$10-8=2$$
$$31+2=33$$
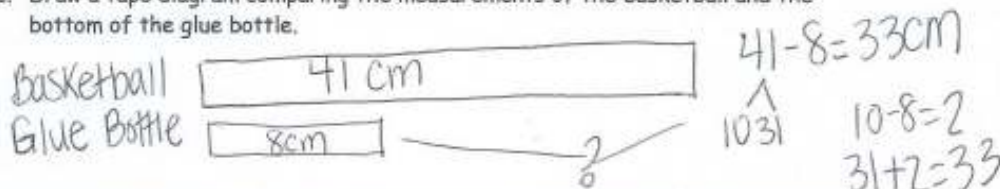
Lesson 10

Objective: Apply conceptual understanding of measurement by solving two-step word problems.

Homework Key

1. Step 1: 20 cm
   Step 2: 49 cm

2. Step 1: Tape diagram is appropriately drawn;
   60 cm
   Step 2: Tape diagram is appropriately drawn;
   101 cm

Work Samples

Use the RDW process to solve. Draw a tape diagram for each step. Problem 1 has been started for you.
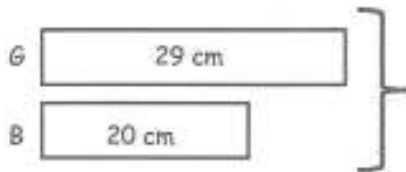
1. There is 29 cm of green ribbon. A blue ribbon is 9 cm shorter than the green ribbon. How long is the blue ribbon?
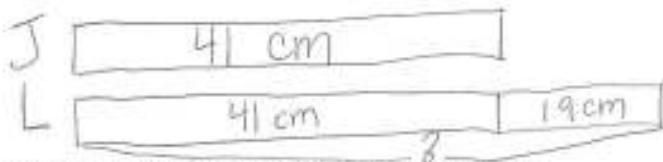
   Step 1: Find the length of blue ribbon.

   G | 29 cm |
   B | ? |
   9 cm
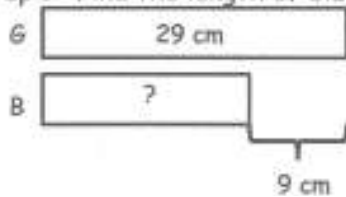
   $29-9=20cm$

   Step 2: Find the length of both the blue and green ribbons.

   G | 29 cm |
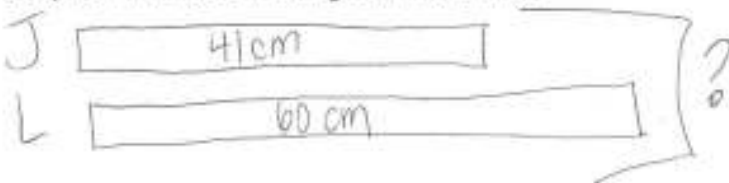   B | 20 cm |
   ?

   $29+20=49cm$

2. Joanna and Lisa drew lines. Joanna's line is 41 cm long. Lisa's line is 19 cm longer than Joanna's. How long are Joanna's and Lisa's lines?

   Step 1: Find the length of Lisa's line.

   J | 41 cm |
   L | 41 cm | 19cm |

   $41+19=60cm$
   $41+9=50$
   $50+10=60$

   Step 2: Find the total length of their lines.

   J | 41cm |
   L | 60 cm |
   ?

   $41+60=101cm$